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# Multiple scattering in polymer dispersed liquid crystal films 

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#### Abstract

A model to describe light transfer in polymer dispersed liquid crystal (PDLC) films has been developed. It takes into account the anisotropy of the nematic liquid crystal droplets, their polydispersity, multiple scattering, and Fresnel reflection at the film interfaces. The model is based on the solution of the radiative transfer equation by the adding method extended for anisotropic liquid crystal droplets. The anomalous diffraction approach was used for the description of light scattering on a single droplet. Calculations of the angular dependence of the intensity of light and contrast ratio on the morphology, illumination and observation conditions of PDLC films in the transmittance mode have been carried out. The results are in qualitative agreement with the published experimental data.


## 1. Introduction

The last decade has seen increasing interest in the study of polymer dispersed liquid crystal (PDLC) films [1] which represent liquid crystal dispersions in a polymer matrix. If there is no applied field (OFF state), the directors of the bipolar nematic droplets are oriented randomly. As a result, the PDLC films give high light scattering. In the applied field, the droplet directors move towards a preferred orientation which results in a decrease in scattering. In a high field (ON state) all the droplet directors are oriented along (in the case of positive anisotropy) the direction of the applied field. Scattering is minimized when the ordinary refractive index of the liquid crystal (LC) matches that of the binder. Owing to this effect and the possibility of making flexible large area structures, PDLC films are a promising material for making displays, switchable windows, and other LC devices.

Most investigations of light scattering in PDLCs are limited by the consideration of a single scattering regime [1]. PDLC films are systems with a high volume concentration of droplets (up to $0.7-0.8$ ) and a high optical density (up to 3-4). To describe light scattering in a PDLC film we must take into account multiple scattering of the light. For such a purpose the well developed radiative transfer theory, based on the radiative transfer equation (RTE), can be used [2-6]. This theory deals with the following characteristics of the medium: scattering and absorption coefficients (determining extinction of the parallel beam of light owing to scattering and absorption

[^0]in the medium) and phase function (determining angular distribution of the scattered light) of a unit volume of the medium.

Classical radiative transfer theory was developed for scattering media with a small concentration of particles, when a regime of independent scattering is realized. To use this theory for a medium with a high concentration of particles, when dependent scattering (the droplets are not in each other's wave zone) is realized, one has to operate with unit volume parameters (scattering and absorption coefficients and phase function), taking into account optical interaction of the scatterers [1, 7-13]. The opportunity to use radiative transfer theory in concentrated dispersions is determined by comparison with the theory of multiple scattering of waves [13] and experiment $[9,10,12]$. For the solution of the radiative transfer theory problems, powerful analytical and numerical methods have been developed [2-6]. Among analytical methods, it is worth paying attention to small angle approximation and diffusion approximation. The latter is used in liquid crystals for the description of light scattering on the thermal director fluctuations [14]. There are many versions and combinations of these methods developed for different scattering problems [5].

In a PDLC film, unit volume parameters are known to depend on the direction of the impinging light. This dependence is taken into account in [11], based on Monte-Carlo simulations. Optical interaction of droplets is considered there by using the structure factor. Light scattering in a medium, where unit volume parameters do not depend on the direction of light propagation, have been investigated in $[15,16]$. The results were
obtained at normal incidence of light. A two-flux approximation of radiative transfer theory has been used to find the maximum value of film reflectance at minimum film thickness in [17]. The authors took into account interface reflectance. It was assumed that the effective refractive index of the matrix depends on the droplet concentration. Scattering in PDLC films with small droplets has been considered in the model of an effective medium [18].

Much work has been done on the problem of single scattering in PDLC films, but investigation of multiple scattering in such films is only beginning. In this work a method for the calculation of the angular distribution of light in PDLC films with spherical droplets under oblique illumination by unpolarized light is developed. The basis of our consideration is the solution of the radiative transfer equation by the adding method, extended by us for PDLC films.

## 2. The model of light propagation in a film

To describe the light intensity in a PDLC film, we used the radiative transfer equation. The scattering characteristics of a medium are: the scattering coefficient $\sigma$, the absorption coefficient $\alpha$, the extinction coefficient $\varepsilon=\sigma+\alpha$ and the phase function. The $\sigma, \alpha$ and $\varepsilon$ coefficients are connected with the scattering $\Sigma_{s}$, absorption $\Sigma_{\alpha}$, and extinction $\Sigma_{\mathrm{e}}$ cross-sections by the relations: $\sigma=N \Sigma_{\mathrm{s}} ; \alpha=N \Sigma_{\mathrm{a}} ; \varepsilon=N \Sigma_{\mathrm{e}}$, where $N$ is the number of droplets per unit volume. The phase function $X(\cos \gamma)$ describes the angular distribution of light scattered by unit volume of the medium; $\gamma$ is the scattering angle such that $\cos \gamma=\mu \mu^{\prime}+\left(1-\mu^{2}\right)^{1 / 2}\left(1-\mu^{\prime 2}\right)^{1 / 2} \cos \left(\varphi-\varphi^{\prime}\right)$, where $\mu=\cos \theta$ and $\mu^{\prime}=\cos \theta^{\prime}$ are the cosines of the axial angles of light scattered and incident, respectively, on the unit volume; $\varphi$ and $\varphi^{\prime}$ are the azimuthal angles of light scattered and incident on the unit volume, respectively. In the OFF state, when the droplet directors are oriented randomly, these characteristics do not depend on the light propagation direction. For such dispersions, reliable methods for the solution of the radiative transfer equation have been developed and the peculiarities of radiative transfer have been well studied. In the ON state, when the droplet directors are oriented in the external field, the elementary volume characteristics depend on the light propagation direction.

Let us consider a layer of bipolar spherical droplets of a liquid crystal in the polymer matrix. Following the authors of $[19,20]$, we consider bipolar droplets, assuming that the director configuration can be modeled as homogeneous $[19,21]$. The film is illuminated at an angle $\theta_{\mathrm{oe}}$ (figure 1 ) from below by an azimuthally symmetrical light beam with intensity $I_{0}$. In the case of an applied electric field, the PDLC film is an anisotropic, axially symmetric system. For such a system the extinction and scattering coefficients depend on the axial


Figure 1. Schematic representation of the scattering geometry (section in the $y z$-plane). Notations are in the text.
angle of light propagation $(\sigma=\sigma(\mu), \varepsilon=\varepsilon(\mu))$; the phase function depends on axial angles and the difference between the incident and scattered azimuthal angles $\left(X=X\left(\mu, \mu^{\prime}, \varphi-\varphi^{\prime}\right)\right)$. The RTE for an anisotropic layer with Fresnel interfaces with the film illuminated by a wide azimuthally symmetrical, unpolarized light beam can be obtained from the general equation [4]. After integration over the azimuth, the RTE can be written in the form:

$$
\begin{align*}
\mu & \frac{\partial I(z, \mu)}{\partial z}+\varepsilon(\mu) I(z, \mu) \\
= & \int_{-1}^{1} X\left(\mu, \mu^{\prime}\right) \sigma\left(\mu^{\prime}\right) I\left(z, \mu^{\prime}\right) \mathrm{d} \mu^{\prime} \\
& +X\left(\mu, \mu_{0}\right) \sigma\left(\mu_{0}\right) I_{0}^{\uparrow} \exp \left[\varepsilon\left(\mu_{0}\right) z / \mu_{0}\right] \\
& +X\left(-\mu \cdot \mu_{0}\right) \sigma\left(\mu_{0}\right) I \downarrow \exp \left[\varepsilon\left(\mu_{0}\right)\left(z_{0}-z\right) / \mu_{0}\right] . \tag{1}
\end{align*}
$$

Here $\mu_{0}=\cos \theta_{0}$ is the cosine of the incident angle inside the film; $z$ is the film depth; $z_{0}$ is the film thickness; $I(z, \mu)$ is the azimuth-averaged intensity of light propagating at depth $z$ at the angle $\arccos \mu ; \varepsilon(\mu)$ and $\sigma(\mu)$ are the extinction and scattering coefficients depending on the direction of the impinging light; $X\left(\mu, \mu^{\prime}\right)=$ $1 / 2 \pi \int_{0}^{2 \pi} X\left(\mu, \mu^{\prime}, \varphi-\varphi^{\prime}\right) \mathrm{d} \varphi$ is the azimuth-average d phase function, called the redistribution function [3], normalized by the condition: $\int_{-1}^{1} X\left(\mu, \mu^{\prime}\right) \mathrm{d} \mu=1 . I_{0}$ and $I \downarrow$ are the intensities of the collimated light beams at the lower and upper interfaces inside the film propagating upwards and downwards.

Normally, the refractive indices of the matrix and glass are close in value. Therefore, we assume that no reflection occurs at the polymer-matrix-glass interface and restrict ourselves to the account of Fresnel reflection only at the air-matrix interface

$$
\begin{align*}
I(\tau=0, \mu) & =\kappa(\mu) I(\tau=0,-\mu)  \tag{2a}\\
I\left(\tau=\tau_{0},-\mu\right) & =\kappa(\mu) I\left(\tau=\tau_{0}, \mu\right) .
\end{align*}
$$

Here the optical depth $\tau=\varepsilon_{\mathrm{m}} z$ and the optical thickness $\tau_{0}=\varepsilon_{\mathrm{m}} z_{0} ; \varepsilon_{\mathrm{m}}=\max \varepsilon(\mu) ; \kappa(\mu)$ stands for the reflection coefficient at the interface [22]:

$$
\kappa(\mu)=\left\{\begin{array}{c}
0.5\left[\frac{t^{2}\left(\theta-\theta_{\mathrm{e}}\right)}{\operatorname{tg}^{2}\left(\theta+\theta_{\mathrm{e}}\right)}+\frac{\sin ^{2}\left(\theta-\theta_{\mathrm{e}}\right)}{\sin ^{2}\left(\theta+\theta_{\mathrm{e}}\right)}\right], \quad \theta<\theta_{\mathrm{r}}  \tag{2b}\\
1, \theta \geqslant \theta_{\mathrm{r}}
\end{array}\right.
$$

where $\theta=\arccos \mu$ is the incident angle inside the film; $\theta_{\mathrm{e}}$ is the angle of refraction and $\theta_{\mathrm{r}}$ is the angle of total internal reflection.

We consider the scattering characteristics of a film in the ON and OFF states. Within the framework of the present paper, we neglect the co-operative effects arising for dense packing of droplets and believe that there is no absorption in liquid crystal droplets, but the matrix can absorb light due to any dye dissolved in it.

In the OFF state, the scattering and absorbing properties of the medium are determined by the phase function $X^{-}(\cos \gamma)$, the scattering coefficient $\sigma^{-}$, and the absorption coefficient $\alpha$. These quantities are related to the scattering characteristics of separate droplets in the following way:

$$
\begin{gather*}
\sigma^{-}=w \sigma_{0}^{-}=w \frac{\bar{\Sigma}_{\mathrm{s}}}{v}  \tag{3}\\
\alpha=(1-w) \alpha_{0}  \tag{4}\\
X^{-}(\theta)=\frac{1}{\bar{\Sigma}_{\mathrm{s}}} \frac{\mathrm{~d} \bar{\Sigma}_{\mathrm{s}}}{\mathrm{~d} \Omega}(\theta) \tag{5}
\end{gather*}
$$

where $v$ is the mean volume of the liquid crystal droplets, $w$ is the volume concentration of droplets, $\bar{\Sigma}_{\mathrm{s}}$ is the scattering cross-section averaged over the sizes and directions of the droplet directors, $\sigma_{0}^{-}=\bar{\Sigma}_{\mathrm{s}} / v$ is the scattering coefficient of the liquid crystal droplets in the OFF state at unit volume concentration, $\mathrm{d} \overline{\bar{\Sigma}_{\mathrm{s}}} / \mathrm{d} \Omega(\theta)$ is the differential cross-section of droplets averaged over the sizes and directions of the droplet directors and $\alpha_{0}$ is the absorption coefficient of the polymer.

In the ON state, the medium is characterized by the azimuth-averag ed phase function $X^{+}\left(\mu, \mu^{\prime}\right)$, the scattering coefficient $\sigma^{+}$and the absorption coefficient $\alpha$.

## 3. Determination of the optical parameters of a unit volume

There is no general theory of scattering by spherical anisotropic particles similar to the Mie theory for isotropic homogeneous particles. The Rayleigh-Gans Approach (RGA) for small droplets and the Anomalous Diffraction Approach (ADA) for large droplets generalized for the case of anisotropic particles are frequently used for the description of scattering and extinction on liquid crystal droplets [19,23]. To determine the unit volume parameters, we used the ADA. The angular distribution of scattered light in this approach is well described in the region of small angles $\left(\gamma \leqslant 30^{\circ}\right)$ and basically cannot be calculated at $\gamma>90^{\circ}$. To give the angular distribution of scattered light in the region of large angles, we extrapolated the size-averaged differential scattering cross-section, found in the ADA, by the exponential dependence. Actually, PDLC dispersions have a size distribution of droplets. Therefore, the scattering characteristics should be averaged over these sizes. We used the log-normal size distribution of droplets [1], and characterized the polydispersity by the mean radius $\bar{R}$ and the variation coefficient $C_{\mathrm{v}}$.

In the OFF state, the angular distribution function of droplet directors is a constant and the averaging over the direction of the droplet directors is reduced to integration over the angular variable. In the ON state, the angular distribution function is described by the Dirac delta function.

We used a spherical coordinate system with the $z$-axis directed perpendicular to the film (figure 1). In this coordinate system, the droplet director is determined by the axial $\theta_{\mathrm{d}}$ and azimuthal $\varphi_{\mathrm{d}}$ angles. The direction of the impinging light prior to scattering is determined by the angles $\theta^{\prime}$ and $\varphi^{\prime}$ and after scattering by the angles $\theta$ and $\varphi$.

In the OFF state, the scattering cross-section averaged over the radii and droplet directors is determined by the following equation:

$$
\begin{align*}
\bar{\Sigma}_{\mathrm{s}}= & \frac{1}{\pi} \int_{0}^{\pi} \int_{0}^{\pi / 2} \int_{R_{1}}^{R_{2}} \Sigma_{\mathrm{s}}\left(\theta_{\mathrm{d}}, \varphi_{\mathrm{d}}, \theta^{\prime}, \varphi^{\prime}, R\right) \\
& \times f(R) \mathrm{d} R \sin \theta_{\mathrm{d}} \mathrm{~d} \theta_{\mathrm{d}} \mathrm{~d} \varphi_{\mathrm{d}} \tag{6}
\end{align*}
$$

where $\Sigma_{\mathrm{s}}\left(\theta_{\mathrm{d}}, \varphi_{\mathrm{d}}, \theta^{\prime}, \varphi^{\prime}, R\right)$ and $\bar{\Sigma}_{\mathrm{s}}$ are the scattering crosssection of the droplet and the averaged scattering crosssection of the droplets, respectively; $R$ is the droplet radius; $f(R)$ is the droplet size distribution function; $R_{1}$ and $R_{2}$ are the minimum and maximum droplet radii. The averaged differential scattering cross-section is:

$$
\begin{align*}
\frac{\mathrm{d} \bar{\Sigma}_{\mathrm{s}}}{\mathrm{~d} \Omega}(\theta)= & \frac{1}{\pi} \int_{0}^{\pi} \int_{0}^{\pi / 2} \int_{R_{1}}^{R_{2}} \frac{\mathrm{~d} \Sigma_{\mathrm{s}}}{\mathrm{~d} \Omega}\left(\theta_{\mathrm{d}}, \varphi_{\mathrm{d}}, \theta, \varphi, \theta^{\prime}=0, \varphi^{\prime}, R\right) \\
& \times f(R) \mathrm{d} R \sin \theta_{\mathrm{d}} \mathrm{~d} \theta_{\mathrm{d}} \mathrm{~d} \varphi_{\mathrm{d}} \tag{7}
\end{align*}
$$

where $\mathrm{d} \Sigma_{\mathrm{s}} / \mathrm{d} \Omega\left(\theta_{\mathrm{d}}, \varphi_{\mathrm{d}}, \theta, \varphi, \theta^{\prime}, \varphi^{\prime}, R\right)$ is the differential scattering cross-section. Note that the averaged differential cross-section is only determined by the scattering angle.

In the ON state we can write the following equations:

$$
\begin{align*}
\bar{\Sigma}_{\mathrm{s}}\left(\theta_{\mathrm{d}}=0, \theta^{\prime}\right)= & \int_{R_{1}}^{R_{2}} \Sigma_{\mathrm{s}}\left(\theta_{\mathrm{d}}=0, \varphi_{\mathrm{d}}, \theta^{\prime}, \varphi^{\prime}, R\right) f(R) \mathrm{d} R \\
\frac{\mathrm{~d} \bar{\Sigma}_{\mathrm{s}}}{\mathrm{~d} \Omega}\left(\theta_{\mathrm{d}}=0, \theta, \theta^{\prime}\right)= & \frac{1}{\pi} \int_{0}^{\pi} \int_{R_{1}}^{R_{2}} \frac{\mathrm{~d} \Sigma_{\mathrm{s}}}{\mathrm{~d} \Omega}\left(\theta_{\mathrm{d}}=0, \varphi_{\mathrm{d}}, \theta, \varphi, \theta^{\prime}, \varphi^{\prime}\right)  \tag{8}\\
& \times f(R) \mathrm{d} R \mathrm{~d} \varphi . \tag{9}
\end{align*}
$$

The scattering cross-section and the differential scattering cross-section are determined by the equations of ref. [19].

Figure 2 shows the calculated angular dependences of the scattering coefficient for non-absorbing liquid crystal droplets in the ON and OFF states. Hereinafter, the calculations are carried out at the light wavelength $\lambda=0.5 \mu \mathrm{~m}$. For PDLC films in the ON state, when the
face, the scattering coefficient has a minimum whose position depends on the ratio between the ordinary $n_{0}$ and extraordinary $n_{\mathrm{e}}$ refractive indices of the liquid crystal and the refractive index of the matrix $n_{\mathrm{m}}$. Depending on the average size, the quantity $\sigma_{0}^{+}\left(\mu^{\prime}\right)$ can decrease mono-


Figure 2. Angular dependences of the scattering coefficients for PDLC films in the ON state (curves 1,3) and in the OFF state (curves 2, 4) at $\bar{R}=0.5 \mu \mathrm{~m}, C_{\mathrm{v}}=0.4$ (curves 1,2 ); and at $\bar{R}=1.2 \mu \mathrm{~m}, C_{\mathrm{v}}=0.4$ (curves 3,4 ); $n_{\mathrm{e}}=1.7, n_{0}=n_{\mathrm{m}}=1.55$.
tonically or increase, pass through a maximum and then decrease. In the OFF state, the value of the scattering coefficient is independent of the light propagation direction. The redistribution function in the ON state also depends on the angle of light incidence on the droplets.

## 4. Light intensity at the air-matrix interfaces

Let us consider a homogeneous layer of a scattering medium. If Fresnel reflection on the interfaces is absent, the luminance factors of reflection $\rho\left(\mu, \mu^{\prime}\right)$ and transmitted $\sigma\left(\mu, \mu^{\prime}\right)$ light are determined by the equations [2-4]:

$$
\begin{align*}
I^{\downarrow}(z=0, \mu)= & \int_{0}^{1} 2 \rho\left(\mu, \mu^{\prime}\right) \mu^{\prime} I_{0}\left(\mu^{\prime}\right) \mathrm{d} \mu^{\prime}  \tag{10}\\
I^{\uparrow}\left(z=z_{0}, \mu\right)= & \exp \left[-\tau_{0} e(\mu) / \mu\right] I_{0}(\mu) \\
& +\int_{0}^{1} 2 \sigma\left(\mu, \mu^{\prime}\right) \mu^{\prime} I_{0}\left(\mu^{\prime}\right) \mathrm{d} \mu^{\prime} . \tag{11}
\end{align*}
$$

Here $I_{0}(\mu)$ is the intensity of light incident on the film. $I^{\downarrow}(z=0, \mu)$ is the back-scattered light intensity at the upper interface; $I\left(z=z_{0}, \mu\right)$ is the forward-scattered light intensity at the film interface. The luminance factor (radiance coefficient) is determined as the ratio of the luminosity at scattering angle $\arccos \mu$ (for illumination of the film interface at angle $\arccos \mu^{\prime}$ ) to the luminosity of a perfectly reflecting white surface [3]. For example, the luminance factor is unity for a white Lambert surface.

To determine the luminance factors $\rho\left(\mu, \mu^{\prime}\right)$ and $\sigma\left(\mu, \mu^{\prime}\right)$ of an anisotropic medium, the calculation technique based on the method of layer doubling has been used. The modification of the method for media with characteristics independent of the light propagation direction is described in [3]. We used the variant of the method which takes into account the medium anisotropy [24].

The luminance factors for the film of doubled thickness are found from the relations which were obtained from the balance equations at the film interfaces. The accuracy of the calculations was checked by comparing the results of the calculation of luminance factors $\rho\left(\mu, \mu^{\prime}\right), \sigma\left(\mu, \mu^{\prime}\right)$ with the data of [3] for homogeneous layers of a medium with the Henyey-Greenstein phase function.

Figure 3 shows the luminance factors at the illumination angle $\arccos (0.7)$ for PDLC films with droplet directors oriented normally to the film interface. The luminance factors of PDLC films with a random orientation of droplet directors are shown in figure 4. For small thicknesses, when the contribution from small angle scattering is high, the luminance coefficient maximum occurs. With an increase in film thickness, the position of this maximum shifts to the normal to the surface. A feature of thick films with oriented directors is the presence of a


Figure 3. Angular dependence of the luminance factor of the PDLC film in the ON state for film illumination at the angle $\arccos (0.7)$. The film is in the immersion liquid and Fresnel reflection at the interfaces is absent. Optical thickness of the film in the OFF state $\tau_{0}^{-}=2$ (curve 1); 4 (2); 8 (3); 16 (4); 32 (5); 64 (6). $\bar{R}=0.2 \mu \mathrm{~m}, C_{\mathrm{v}}=0.4$, $n_{\mathrm{e}}=1.7, n_{0}=n_{\mathrm{m}}=1.55$.
marked maximum in the angular dependence of the scattered light intensity. This is because of the minimum in the scattering coefficient of oriented droplets. With an increase in optical thickness this maximum becomes more pronounced as opposed to the case of disoriented droplets, where, with increased thickness, the maximum becomes less pronounced and the extremum disappears quickly. The calculated angular distributions at different thicknesses (figure 4) agree with the experimental data [16].

The angular dependence of direct (coherent) transmittance $T=\exp \left(-w \sigma_{0} z_{0} / \cos \theta_{0}\right)$ is shown in figure 5 ; here $n_{0}<n_{\mathrm{m}}$. There is a maximum for this dependence; it takes place at the angle where the effective refractive index of the droplet is equal to that of the polymer. This maximum was found experimentally [25, 26]. Note the non-monotonic change of transmittance at oblique angles of observation for large droplet sizes. Under certain conditions, it is possible to get an additional maximum. This maximum arises from the effect of specific changes in the scattering coefficient on the direction of the impinging light. It can be observed when the film is in an immersion medium, where the refraction at the film interface is negligible.

Having found the luminance factors of a homogeneous layer with no Fresnel reflection at the interfaces, we turn


Figure 4. Angular dependence of the luminance factor of the PDLC film in the OFF state for film illumination at the angle $\arccos (0.7)$. The film is in the immersion liquid and Fresnel reflection at the interfaces is absent. Optical thickness of the film in the OFF state $\tau_{0}^{-}=1$ (curve 1); 2 (2); 4 (3); 8 (4); 16 (5); 32 (6). $\bar{R}=0.2 \mu \mathrm{~m}, C_{\mathrm{v}}=0.4$, $n_{\mathrm{e}}=1.7, n_{0}=n_{\mathrm{m}}=1.55$.
to the solution of the problem of determining the light intensity at the film interfaces with regard to Fresnel reflection. To solve this problem one has to use the balance equations for the direct and diffuse components of light at the film interfaces. The procedure is similar to the one described in [24]. Here we write only the equation for direct transmittance $T$ at oblique illumination,

$$
\begin{equation*}
T\left(\mu_{0}\right)=\frac{\left[1-\kappa\left(\mu_{0}\right)\right]^{2} \exp \left[-\tau_{0} e\left(\mu_{0}\right) / \mu_{0}\right]}{1-\kappa^{2}\left(\mu_{0}\right) \exp \left[-2 \tau_{0} e\left(\mu_{0}\right) / \mu_{0}\right]} \tag{12}
\end{equation*}
$$

where $\kappa\left(\mu_{0}\right)$ is determined by equation (2b) for $\mu=\mu_{0}=$ $\cos \theta_{0} . \theta_{0}$ stands for the incidence angle inside the film, $n_{\mathrm{m}} \sin \theta_{0}=\sin \theta_{\mathrm{e}}=\sin \theta_{\mathrm{oe}} ; e\left(\mu_{0}\right)=\varepsilon\left(\mu_{0}\right) / \varepsilon_{\mathrm{m}}$. At normal illumination this equation coincides with that deduced in [27].

The angular dependences of the transmitted light (normalized to the intensity of the incident light) for diffuse illumination of the film in the OFF state are monotonic. The results for the ON state are shown in figure 6 . There are two sets of curves in the figure; one set is for directly transmitted light, the other is for the scattered light. The intensity of the directly transmitted light decreases monotonically with the observation angle. The thicker the film the more extended the dependence of the directly transmitted light, so that maximum transmittance takes place at $\theta_{\mathrm{e}}=0^{\circ}$. The intensity of scattered


Figure 5. Angular dependenœ of the direct transmittance $T=\exp \left(-w \sigma_{0} z_{0} / \cos \theta_{0}\right) . \bar{R}=0.3 \mu \mathrm{~m}$ (curve 1); $0.6 \mu \mathrm{~m}(2) ;$ $1.5 \mu \mathrm{~m}$ (3); $1.9 \mu \mathrm{~m}$ (4). $C_{\mathrm{v}}=0.2, \quad n_{\mathrm{e}}=1.7, \quad n_{0}=1.52$, $n_{\mathrm{m}}=1.55, w z_{0}=1 \mu \mathrm{~m}$. Fresnel reflection is suppressed by using immersion liquid.


Figure 6. Angular dependenœ of the directly transmitted (curves 1-4) and scattered light (curves $1^{\prime}-4^{\prime}$ ) with diffuse illumination of the film in ON state for $w z_{0}=5 \mu \mathrm{~m}$ (curve 1, 1'), $10 \mu \mathrm{~m}\left(2,2^{\prime}\right), 20 \mu \mathrm{~m}\left(3,3^{\prime}\right), 30 \mu \mathrm{~m}\left(4,4^{\prime}\right)$. $\bar{R}=0.25 \mu \mathrm{~m}, C_{\mathrm{v}}=0.2, n_{\mathrm{e}}=1.7, n_{0}=n_{\mathrm{m}}=1.52$.
light has a maximum at $\theta_{\mathrm{e}}=90^{\circ}$, when the thickness is small; with increasing thickness this maximum shifts to smaller values of $\theta_{\mathrm{e}}$. These results help us to understand the process of angular distribution formation and to find conditions for the decrease of haze.

## 5. Contrast ratio

By the contrast ratio $(C R)$ is normally meant the value of the ratio of the light intensity collected on the photodetector from the PDLC film in the ON and OFF states. $C R$ depends on the film's properties, the film illumination method, and on the collection angle. We consider the $C R$ of a film illuminated by a beam of light azimuthally symmetric about a cone of revolution with the angle $\theta_{0 \mathrm{e}}$ (figure 1). A receiver collects the transmitted light under the angle $\theta_{\mathrm{e}}=\theta_{0 \mathrm{e}}$.

The constrast ratio at small optical thicknesses (figure 7) can be estimated by the single scattering approximation. Multiply scattered light decreases $C R$. Qualitative estimation of the influence of scattered light for thin and thick films, as well as the influence of the collection angle, can be made for practical situations using the above results.

Figure 8 shows the dependence of $C R / C R_{\max }\left(C R_{\max }\right.$ stands for maximum values of $C R$ at the corresponding collection angle) on the droplet radius for normal illumination and a constant volume of droplets in the film. It illustrates the situation when the receiver collects


Figure 7. Dependence of $\lg \mathrm{CR}$ on the optical thickness $\tau_{0}^{-}$ (in the OFF state) for $\bar{R}=0.2 \mathrm{~mm}, C_{\mathrm{v}}=0.4, n_{\mathrm{e}}=1.7$, $n_{0}=n_{\mathrm{m}}=1.55$ at $\Delta \theta_{\mathrm{c}}=0.0175 \mathrm{rad} . \theta_{0 \mathrm{e}}=6.81^{\circ}$ (curves 1,2 ), $\theta_{0 \mathrm{e}}=22.4^{\circ}$ (curves 3,4 ). The results in curves 2 and 4 are obtained with regard paid to the multiple scattered light.


Figure 8. Dependence of $C R / C R_{\max }$ on the droplet radius for normal illumination of the film by a parallel beam of light and for the collection angle $0.2^{\circ}$ (curve 1 ), $4^{\circ}(2), 10^{\circ}(3)$, $20^{\circ}(4), 40^{\circ}(5) . w z_{0}=10 \mu \mathrm{~m}, n_{\mathrm{e}}=1.7, n_{0}=n_{\mathrm{m}}=1.55$.
all the directly transmitted light and part of the scattered light. The latter is determined by the collection angle $\theta_{\mathrm{c}}$. With an increase in collection angle, the position of the maximum $R_{\mathrm{m}}$ on the dependence $C R-R$ shifts to smaller sizes. At the chosen values of parameters $w z_{0}$, refractive indices, and collection angles, the value of $R_{\mathrm{m}}$ shifts from $R_{\mathrm{m}} \approx 0.75 \mu \mathrm{~m}$ to $R_{\mathrm{m}} \approx 0.2 \mu \mathrm{~m}$. The shift of $R_{\mathrm{m}}$ to smaller values with increase in the collection angle is due to a rise in the amount of scattered light. With a decrease in the collection angle the $R_{\mathrm{m}}$ value tends to the maximum dependence of the extinction efficiency factor on droplet size for the film in OFF state.

The above results are in good agreement with the known experimental data published in $[15,16,20,25,26]$. The model correctly describes the main features of $C R$ behaviour with change in the morphology of the PDLC film. It can be used for the theoretical estimation of the transmitted and scattered light under different conditions of illumination and observation of the film.

## 6. Conclusion

A model for the calculation of radiative transfer in PDLC films with regard paid to anisotropy of the nematic liquid crystal droplets, the polydispersity of the droplets, multiple scattering, and Fresnel reflection at the film interfaces has been developed. The model is based on the integro-differential radiative transfer equation. It can be used for a wide range of optical thicknesses of film, from small values when we can restrict ourselves to the
single-scattering approximation, to large values where multiple scattering of light has to be taken into account. Moreover, it can be used for other types of composite LC film [28, 29].

The model allows one to analyse the angular characteristics of reflected and transmitted light; and the contrast ratio dependence on illumination conditions, collection angles, spectral range of the incident light, size and concentration of the droplets, the value of the applied field, and the director field configuration in the droplet. The proposed model is flexible and makes possible detailed observations concerning the problems of optimization of contrast ratio, angular structure, transmittance, and viewing angle of PDLC films. Further research on this problem is being undertaken.

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